Symbol Detection Methods for DFEs in Trellis Coded Modulation Systems

격자코드 변조 시스템에서 DFE의 심볼판정 알고리즘 제안

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ABSTRACT

In this paper, we present symbol detection methods for decision feedback equalizers (DFE) in trellis coded modulation systems. The proposed symbol detectors improve symbol error rate (SER) by exploiting the coding structure of trellis coded modulation (TCM). For example, for 8-PAM signals the achieved SER with the proposed detection scheme is improved to 2×10^{-5} from 2.5×10^{-2} of the conventional symbol-by-symbol detector under AWGN channel at 20dB SNR. This SER improvements mitigate error propagation of DFE.and produces significant over-all SER improvement for under multipath channels (for example, from 0.26 to 0.01 and 0.005 under a severe multipath channel 20dB SNR as shown in the simulation result of this paper).

요 약

이 논문은 Trellis Coded Modulation 시스템에서 궤한 채널 등화기의 성능 향상을 위한 효율적인 심볼 판정 알고리듬을 제안한다. 제안된 심벌 판정기는 Trellis 코드의 구조를 이용하여 심벌 에러율 을 향상시킨다. 예컨대 8-PAM시그널의 경우 20dB SNR에서 기존 강제 심볼 판정기의 에러율 2.5×10^{-2}에서 2×10^{-5}으로 향상되었다. 이런 판정기의 심벌 에러율 의 개선은 다중경로채널에서의 DFE의 심벌 에러율을 성능을 향상시키는데 본 논문의 시뮬레이션 결과 심벌 에러율 0.26에서 0.01 과 0.005 으로 개선되었음을 확인하였다

key Words : Decision feedback equalizer, Symbol error rate, Error propagation, Sequence maximum likelihood detection, Viterbi Algorithm

Ⅰ. Intorduction

The decision feedback equalizer (DFE) has been recognized as a strong candidate to combat severe multipath channels with long delay spread in wireless digital broadcasting systems. The DFE does not suffer from noise enhancement, thus outperforms linear equalizers. Furthermore, DFE achieves near Maximum-Likelihood performance with a simple structure, in contrast to sequence detector whose complexity grows exponentially as

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channel length increases. However, the recursive structure of DFE is susceptible to decision errors; a single symbol error in the DFE system can trigger a sequence of decision errors, known as error propagation. Hence, DFE requires reliable delay-free symbol decisions, and this drawback has restricted wide use of DFE, especially, in the low SNR environment.

Contrast to various attempts to incorporate DFE into trellis decoding to simplify MLSE Viterbi algorithm [1], [2] little efforts are made to utilize trellis codes to mitigate error propagation, frustrated by the decoding delay. Among those few efforts, authors in [3], [4], [5] devised a simple way to utilize tentative state information from the Viterbi TCM decoder for symbol detection. For each equalizer output, the slicer is optimized with respect to the signal coset corresponding to the state estimation from Viterbi TCM decoder.

In this paper, we generalize the approaches in [3], [4], [5] as a maximum likelihood detection based on truncated trellis. We present two detection schemes named maximum likelihood trellis symbol detector (MLTSD) and extended maximum likelihood trellis symbol detector (EMLTSD) for DFE. The proposed methods can be considered as a maximum likelihood (ML) detection based on the current and past received signals, while the conventional Viterbi TCM decoder can be viewed as an ML detector based on the entire transmitted signals.

In Section II we propose two detection schemes based on trellis structure of the received TCM signals, named MLTSD and EMLTSD. In Section III theoretical analysis of the symbol error rate of MLTSD is given. We develop a SER formular of the proposed MLTSD for general cases including low SNR. However, in order to obtain a simple and intuitive SER expression, we derive a high SNR approximation for MLTSD. In Section IV simulation results are presented for the TCM system used in ATSC DTV standard. The theoretical SER prediction is compared with the simulated SER and performance enhancement for DFE is presented. Section V concludes.

II. Proposed Detection Schemes

We consider a trellis coded modulation system over an AWGN channel.

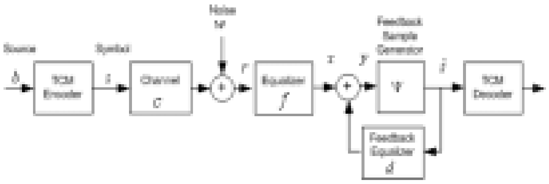

Figure 1. Trellis Coded Communication System with DFE

Let r_k denote the received signal and s_k denote the original source signal corresponding to r_k drawn from a constellation set A.

The conventional slicer is the symbol-by-symbol maximum-likelihood detector,

$$s_k^{ML} = \arg\max_{s\in A} P(r_k|s), \tag{1}$$

which is given by the following under AWGN

$$s_k^{ML} = \arg\max_{s\in A} \| r_k - s \|^2. \tag{2}$$

In a TCM system, it is natural to consider the past N-1 received signals as well, since only certain patterns of received symbols are allowed in TCM. Hence, the enhanced ML detection for a TCM system would be (denoted by $\widehat{s_k}$ and named Maximum-Liklihood Trellis Symbol Detecor (MLTSD))

$$\widehat{s_k} = \pi_0(\arg\min_{s\in T^N} \| \mathrm{r}_k - \mathrm{s}_k \|^2), \tag{3}$$

where $\mathrm{r}_k = [r_k, \cdots, r_{k-N+1}]$, the vector consists of past N symbols, and $\mathrm{s}_k = [s_k, \cdots, s_{k-N+1}]$, is the original symbol sequence belong to the set of all possible TCM symbol patterns of length N, denoted by T^N. π_0 denotes the projection operator taking 0-th element of a vector, i.e., $\pi_0(\mathrm{r}_k) = r_k$.

The performance of this detection depends on the choice of N and it degenerates into the conventional symbol-by-symbol slicer when $N=1$.

More generally, one can extract M symbols from the length N-ML sequence detection (Extended MLTSD, or EMLTSD)

$$[\widehat{s_k^k}, \cdots, \widehat{s_{k-M+1}^k}] = \pi_{0,\cdots M-1}(\arg\min_{s \in T^N} \| r^k - s \|^2), \quad (4)$$

where $\widehat{s_m^k}$ means the m-th estimated symbol at the time k based on the current and past received signals $r_k = [r_k, \cdots, r_{k-N+1}]$.

Notice that the performance of these detection schemes cannot be better than that of Maximum-Likelihood Trellis Decoder. Hence, these detection schemes are redundant for a communication system equipped only with a linear equalizer. However, in the presence of DFE, these detection schemes can be used as feedback symbol generator which can mitigate error propagation due to the false decision error of DFE, and produces better equalization performance.

In a DFE system, the equalizer output at time k, y_k is given as sum of the feedforward filter output x_k and the feedback filter output z_k, and

$$z_k = \sum_{n=1}^{N_d} d_n \Psi(y_{k-n}), \quad (5)$$

where $\{d_k | k=1, \ldots, N_d\}$ denotes feedback equalizer coefficients and $\Psi(y_k)$ denotes the feedback sample generated from y_k. In a conventional DFE system the feedback sample is generated by a slicer, the symbol-by-symbol ML detector in (2).

In a TCM system, we can use the MLTSD output, $\widehat{s_k}$

$$z_k = \sum_{n=1}^{N_d} d_n \widehat{s_{k-n}}, \quad (6)$$

or EMLTSD output,

$$z_k = \sum_{n=1}^{M} d_n \widehat{s_{k-n}^k} + \sum_{n=M+1}^{N_d} d_n \widehat{s_{k-n}^{k+M-n}}, \quad (7)$$

With this M-tuple estimation in EMLTSD the first M inputs to the feedback filter of the DFE are refreshed at each iteration, while the rest inputs are obtained via delay as in a conventional DFE.

The proposed symbol detection methods (3) and (4) can be implemented by the Viterbi algorithm. In fact, the TCM Viterbi decoder with the decoding depth $\Gamma=1$ with a modification to output symbols (instead of bits) will achieve the proposed symbol detection for arbitrary infinitely long N.

For MLTSD, the generated feedback symbol might be wrong and can be corrected by the TCM decoder later (after decoding depth). In the EMLTSD case, after-detection correction can be avoided by setting M to the decoding depth Γ which is usually set to the 5 times of constraint length of the encoder convolution code. In that case the M unreliable inputs to the DFE feedback filter are updated every iteration, while reliable TCM output after Γ symbols are reused via delay.

III. Performance Analysis

In this section we investigate the performance of MLTSD and EMLTSD. The complete performance analysis should theoretically address the bit error rate (BER) improvement of TCM decoder due to MLTSD and EMLTSD. However, it is not an easy task because the error propagation of DFE is difficult to model. Hence, in this paper analysis is focused on symbol error rate (SER) instead of BER. We search for a closed expression for SER of MLTSD, and compare SER performance of MLTSD and the conventional symbol-by-symbol ML detector.

Let P_s denote the symbol error rate of MLSTD. To determine the bound of P_s, we need to modify the conventional node error bound for TCM

The symbol error of MLTSD occur when a symbol sequence reached to a wrong state has the best metric or wrong symbol is detected in a correct state (error in the parallel transition) Assuming a sequence of signals C is transmitted, let $P(C\rightarrow C_L^{\tau})$ denote the probability of the following event: A sequence C_L^{τ} that diverges from C at time $t-\tau$ and still unmerged at time t has the best metric at t. Then, by Union Bound theorem,

$$P_s \le \sum_{\tau=0}^{\infty}\sum_{C}\sum_{L} P(C)P(C\rightarrow C_L^{\tau}). \tag{8}$$

This bound can be further refined by modifying generation function as done in [6] for the truncation error probability bound for the Viterbi convolution decoder. However, obtaining the upper bound (8) is specific to the generating polynomial of a specific trellis code. To obtain a general SER formular, in this paper we consider SER approximation for high SNR.

Define a minimum distance d_f as the following

$$d_f^2 = \min_{C,\tau} \| C - C_L^{\tau} \|^2. \tag{9}$$

Note that

$$d_{\min} \le d_f \le d_{free}, \tag{10}$$

where $d_{\min}$ is the minimum distance among the constellations (un-coded minimum distance)

$$d_{\min}^2 = \min_{s_k, s_l \in A} \| s_k - s_l \|^2, \tag{11}$$

and d_{free} is the minimum distance among trellis sequences starting from the zero state and returning to the zero state [7],

$$d_{free}^2 = \min_{C_k, C_l \in T} \| C_k - C_l \|^2. \tag{12}$$

For sufficiently high SNR, we assume that only the closest sequences to the transmitted one have any significant error probability as assumed to get a high SNR approximation of node error bound [7]. Then, we have

$$P_s \approx N(d_f) Q\left(\sqrt{\frac{d_f^2 E_s}{2N_0}}\right) \tag{13}$$

where $N(d_f)$ is the averaged number of symbols at time t for the sequences that are distance d_f from the transmitted sequence. For example, consider a TCM system for 8-PAM given in Figure 2.

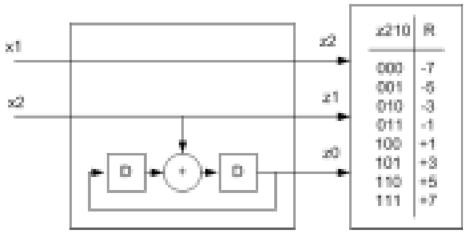

Figure 2. Trellis Coded Modulation Encoder System for 8PAM.

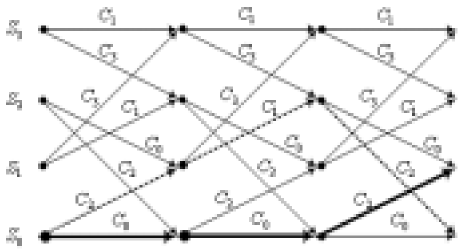

Figure 3. Trellis Diagram for the Encoder given in Figure 2.

The trellis diagram for this TCM system is given by Figure 3, where the signal cosets are

$$C_0=(-7,1),\ C_1=(3,-5),\ C_2=(-3,5),\ C_3=(7,-1) \tag{14}$$

In the trellis diagram the dotted sequence is the d_{free}-distance sequence from the sequence all zeros are transmitted and the bold sequence is the d_f-distance from the all-zero sequence. Hence, in this example, we have

$$d_{\min} = 2 \quad (15)$$
$$d_f = \sqrt{d^2_{\min}(C_0, C_2)} = 4$$
$$d_{free} = \sqrt{2d^2_{\min}(C_{0,} C_2) + d^2_{\min}(C_0, C_3)} = 6$$

Furthermore, we have $N(d_f)=2$, since each trellis in Figure 3 (or each coset) contains 2 symbols. Hence,

$$P_s \approx 2Q\left(4\sqrt{\frac{E_s}{2N_0}}\right) \quad (16)$$

Ⅳ. Simulation Results

Figure 4 shows the performance of MLTSD under AWGN channel for the 8-PAM TCM defined in Figure 2, which is used in American HDTV (ATSC) system for 8-VSB system [8].

The SER performance of MLTSD is substantially improved in comparison with the symbol-by-symbol detection, and lower bounded by that of the Viterbi TCM decoder. For high SNR, the SER simulation result confirms the validity of theoretical prediction in (16).

Figure 5 presents SER performance of DFE equipped with MLTSD and EMLTSD (M=5) over a multipath channel

$$c = [1, 0.1, -0.2, 0.1, 0.20.7] \quad (17)$$

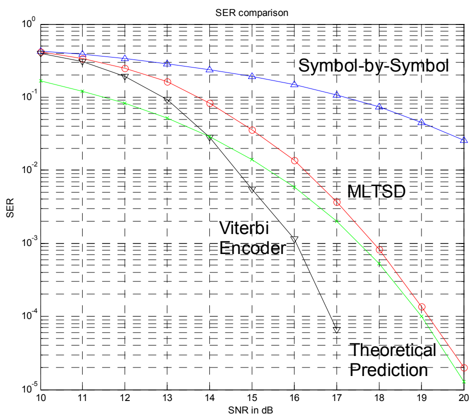

Figure 4. SER improvement of MLTSD over AWGN channel.

The DFE with the proposed MLTSD and EMLTSD without TCM decoder outperforms the DFE with symbol-by-symbol detector. Futhremore, the DFE alone with the proposed schemes outperforms TCM decoder after the DFE with the symbol-by-symbol detector. Hence, The SER performance can be further improved when the output of DFE with MLTSD or EMLTSD is processed by TCM decoder again.

Ⅴ. Conclusion

In this paper, we have proposed symbol detection schemes, MLTSD and EMLTSD, to improve SER and, consequently, enhance DFE performance by exploiting the coded modulation structure. We analyzed SER performance of MLTSD for high SNR. Simulation results confirmed improved SER performance of MLTSD for the AWGN chanel and enhanced DFE performance with MLTSD and EMLTSD.

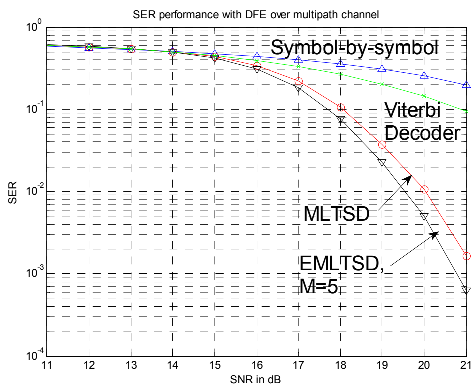

Figure 5. SER improvement of DFE with MLTSD and EMLTSD over a multipath channel.

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