

# Improved Iterative Decoding of Parallel and Serially Concatenated Trellis Coded Modulation

## 병렬 및 직렬적으로 연결된 트렐리스 부호화 변조 기법을 위한 향상된 반복적 복호 기법

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### Abstract

For parallel and serially concatenated trellis coded modulation (TCM), improved iterative decoding schemes with a simple mechanism are proposed and their performances are compared with those of conventional decoding schemes. Simulation results have shown that the proposed schemes have provided a considerable decoding gain in additive white Gaussian noise (AWGN) channels and Rayleigh fading channels, even if they can be implemented by a simple modification of conventional decoding algorithms.

### 요약

본 논문에서는, 병렬 혹은 직렬적으로 연결된 트렐리스 부호화 변조 기법 (Trellis coded modulation: TCM)을 위한 간단한 구조를 가진 향상된 반복적 복호 기법들이 제안되며, 동시에 제안된 기법들의 성능을 기존 기법들과 비교 제시한다. 제안된 복호 알고리즘은 기존 알고리즘의 단순 변형을 통해서 구현될 수 있음에도 불구하고, 모의 실험 결과는 제안된 기법들이 부가 백색 가우스 잡음 채널 (Additive white Gaussian noise channel: AWGN channel) 및 레일리 (Rayleigh) 페이딩 채널 상에서 상당한 부호 이득을 제공함을 보여 준다.

*Key words* : Iterative decoding, Bit-interleaved, Parallel concatenated code, Serially concatenated code, Trellis coded modulation

## 1. Introduction

As the near-capacity gains claimed for parallel concatenated convolutional codes (PCCC's), what are called as Turbo codes, and serially concatenated convolutional codes (SCCC's) have been confirmed and widely reported in the literature, the range of applications of PCCC's and

SCCC's has expanded to many areas of communications. Especially, it is interesting to combine PCCC's or SCCC's with a bandwidth-efficient modulation in order to improve the transmission spectral efficiency. Many papers have shown the good results of applying PCCC's or SCCC's to trellis-coded modulation (TCM) [1, 2].

In this paper, we propose the enhanced iterative decoding schemes for parallel and serially concatenated trellis coded modulation (PC-TCM and SC-TCM) with an interleaver between an encoder and a modulator. By the soft or hard decision feedback according to the new iterative decoding algorithm, the proposed schemes use the sequences of probability distributions that have not

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$f(X_i, j) \in \{1, 0\}$  is the value of the  $j$ -th bit of the label for  $X_i$ . The initial value of  $P(v_i^j; I)$ —prior to any decoding—is assumed to be constant for all  $i$ .

Note that the probability  $P(X_i)$  is assumed to be equal for any  $X_i \in \mathcal{X}(i, a)$  in the first decoding step.

## 2. Algorithm Using Hard Decision Value

For each received signal  $Y_t$ , BBC using hard-decision feedback calculates the probabilities

$$\begin{aligned} P(v_i^j = a; O) &= \frac{P(v_i^j = a | Y_t)}{P(v_i^j = a; I)} \\ &= \sum_{X_i \in \mathcal{X}(i, a)} \{P(Y_t | X_i, \rho_t) P(X_i)\} \end{aligned} \quad (5)$$

where

$$P(X_i) = \begin{cases} 1, & \text{if } f(X_i, i) = a \text{ and } f(X_i, j) = \hat{v}_i^j \text{ (for all } j \neq i) \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

where  $\hat{v}_i^j$  is the previous iterative decoding decision.

The probability  $P(v_i^j; O)$  calculated by (3) or (5) is deinterleaved and used by a conventional SCCC-decoder using the SISO algorithm [3]. Then,  $P(C_i; O)$  is interleaved and fed back for the next iteration.

## 3. Computation of Input and Output Bit Computation

In this subsection, the operation of the metric converters shown in Fig. 2 is described. Consider a rate  $k_o/n_o$  trellis encoder such that each input symbol  $U$  consists of  $k_o$  bits and each output symbol  $C$  consists of  $n_o$  bits.

Bit-to-symbol metric converter calculates the symbol probabilities

$$P(c; I) = \prod_{j=1}^{n_o} P_j(c^j; I) \quad (7)$$

$$P(u; I) = \prod_{j=1}^{k_o} P_j(u^j; I) \quad (8)$$

where  $c^j$  denotes the value of the  $j$ -th bit of the coded symbol  $C_k=c$ ;  $j=1, \dots, n_o$  and  $u^j$  denotes the value of the  $j$ -th bit of the input symbol  $U_k=u$ ;  $j=1, \dots, k_o$ .

Symbol-to-bit metric converter calculates the bit probabilities

$$P_j(c^j; O) = H_{c^j} \sum_{c: c^j = c^j} P(c; O) \prod_{\substack{i=1 \\ i \neq j}}^{n_o} P_i(c^i; I) \quad (9)$$

$$P_j(u^j; O) = H_{u^j} \sum_{u: u^j = u^j} P(u; O) \prod_{\substack{i=1 \\ i \neq j}}^{k_o} P_i(u^i; I) \quad (10)$$

where  $H_{c^j}$  and  $H_{u^j}$  are normalization constants such that

$$H_{c^j} = \sum_{c^j \in \{0,1\}} P_j(c^j; O) = 1 \quad (11)$$

$$H_{u^j} = \sum_{u^j \in \{0,1\}} P_j(u^j; O) = 1 \quad (12)$$

## 4. Description for PC-TCM

We consider the association of  $2^2$ -ary QAM or PSK modulation and encoder built from a standard PCCC by using puncturing technique as depicted in Fig. 3. In order to obtain symbols affected with uncorrelated noises at the decoder input and to randomize the data prior to modulation to limit the peak to average ratio of the envelope of the modulated waveform, a random interleaver  $\pi_1$  is inserted between the encoder and the modulator.

The new iterative decoding scheme for PC-TCM is presented in Fig. 4, where B-to-S indicates bit-to-symbol metric converters. Each SISO module is a four-port device that accepts at the input the sequences of probability distributions  $\{P(C; I), P(U; I)\}$  and outputs the sequences of probability distributions  $\{P(C; O), P(U; O)\}$  [3]. The proposed schemes are identical with conventional decoder schemes using the SISO module. However, there is one important difference. In these schemes, the sequences of probability distributions  $\{P(C_i; O),$

$P(C_2; O)$ ,  $P(C_i; O)$ ) that have not been utilized in conventional PCCC-decoder and SCCC-decoder are iteratively fed back by B-Module. These sequences can be iteratively fed back without amplification of self-information because the interleaver  $\pi_1$  is inserted between the encoder and the modulator.

The decoding steps using hard decision values are as follows. For each received signal  $Y_t$ , BBC with hard-decision feedback calculates the probabilities defined by Eq. (5). In Eq. (6),  $f(X_t, j) \in \{1, 0\}$  is the value of the  $j$ -th bit of the label for  $X_t$  and  $\hat{v}_t^j$  is the previous iterative decoding decision. The probability  $P(X_t)$  is assumed to be equal for any  $X_t \in \mathcal{X}(i, a)$  in the first decoding step.

Finally, the probability  $P(v_t^i; O)$  calculated by Eq. (3) is deinterleaved and used by a conventional PCCC or SCCC decoder using the SISO algorithm [3]. Then,  $\{P(C_1; O), P(C_2; O)\}$  generated by the SISO modules are interleaved and fed back into BBC for the next iteration.

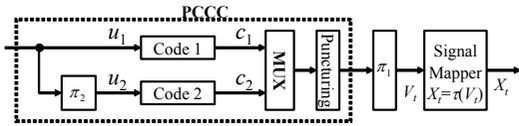


Fig. 3. Encoder/modulator block diagrams of PC-TCM.

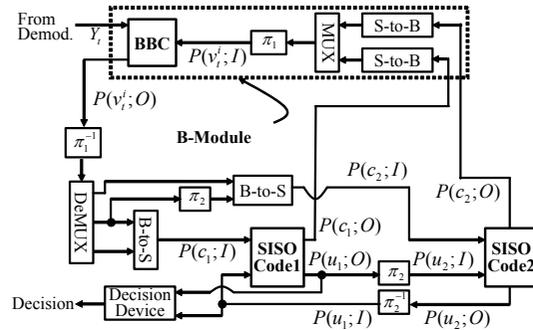


Fig. 4. Iterative decoding scheme for PC-TCM

### III. Experimental Results and Discussion

#### 1. Performance of SC-TCM

To show the performance of the SC-TCM

decoded using the new algorithm, we have simulated a rate  $R=1/3$  SCCC and 8PSK modulation, joined by a random interleaver  $\pi_1$ . To achieve the best performance, the gray mapping (see Fig. 5) and soft-decision feedback are used.

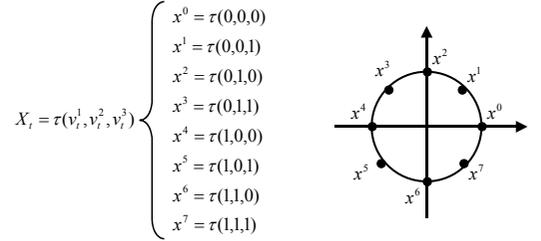


Fig. 5. The gray mapping used in simulation for 8 PSK modulation

The SCCC is formed by an outer code with rate 2/3 obtained by puncturing a systematic, recursive, rate 1/2 convolutional code with generating matrix

$$G(Z) = \left( 1, \frac{1+Z^2}{1+Z+Z^2} \right) \quad (13)$$

(the rate 2/3 is obtained by puncturing every other parity-check bit), and an inner code consisting of a rate 1/2 systematic recursive convolutional code with the same previous generating matrix, joined by a random interleaver  $\pi_2$ .

We have computed the bit error rate (BER) using the Monte Carlo method as a function of  $E_b/N_0$ . In Figs. 6–8, the results over Gaussian channels are plotted for information block length 318, 596, and 700. In these figures, [BI-On, FB-On], [BI-On, FB-Off], and [BI-Off, FB-off] means the proposal method ( $\pi_1$  is used in Fig. 1 and the proposed feedback is also used), the conventional method-A ( $\pi_1$  is used and the proposed feedback is not used), and the conventional method-B ( $\pi_1$  is not used and the proposed feedback is not used), respectively. Also, BI, FB and IT mean bit-interleaver, feedback, iteration number, respectively.

Fig. 9 shows that BER comparison between SCCC-TCM and the 64-state TCM by Ungerboeck

under additive white Gaussian noise (AWGN) channel.

We have also simulated rates  $R=1/4$  and  $1/4$  SCCC and 8PSK modulation as shown in Fig. 10. In this case, the SCCC is formed by an outer code that is the systematic, recursive, rate  $1/2$  convolutional code with generating matrix Eq. (13), and an inner code consisting of a rate  $1/2$  systematic recursive convolutional code with the same previous generating matrix, joined by a random interleaver ( $\pi_2$ ).

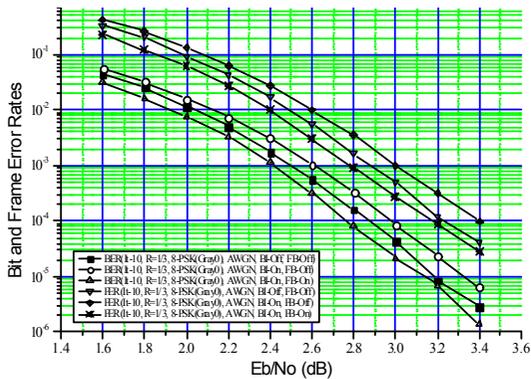


Fig. 6. BER and FER Comparison for SCCC-TCM under AWGN Channel. Data Block Size: 318, Overall Code Rate: 1/3

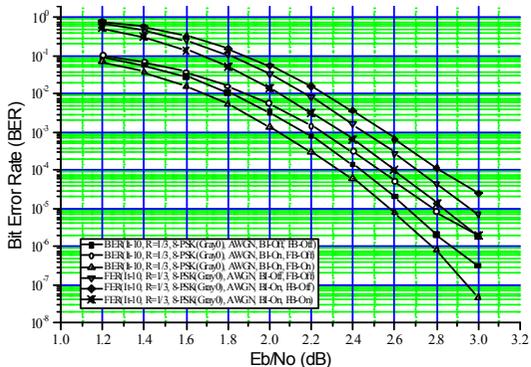


Fig. 7. BER and FER Comparison for SCCC-TCM under AWGN Channel. Data Block Size: 596, Overall Code Rate: 1/3

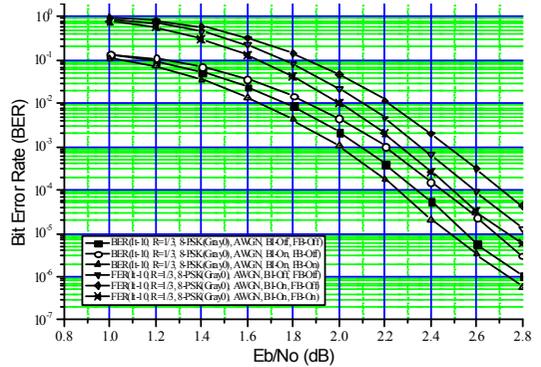


Fig. 8. BER and FER Comparison for SCCC-TCM under AWGN Channel. Data Block Size: 700, Overall Code Rate: 1/3

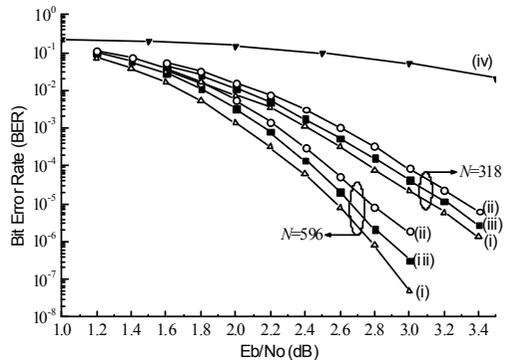


Fig. 9. BER Comparison for SCCC-TCM under AWGN channel. Overall Code Rate: 1/3. (i) Proposal [BI-On, FB-ON], (ii) Conventional-A [BI-On, FB-Off], (iii) Conventional-B [BI-Off, FB-Off], (iv) Conventional TCM (64-State)

The performance comparison over a Rayleigh fading channels is also presented in Fig. 11. From this figure, we can see that over a Rayleigh channel, the interleaver  $\pi_1$  is necessary to achieve the best performance.

## 2. Performance of PC-TCM

To show the performance of the proposed PC-TCM schemes, we have simulated a rate  $1/3$  PCCC with 8-PSK modulation and a random interleaver  $\pi_1$ . To achieve the best performance, the gray mapping and the soft-decision feedback are

also used. Two equal 8-state rate 1/2 recursive systematic convolutional codes are used to form the PCCC and their generating matrix is

$$G(Z) = \left( 1, \frac{1 + Z + Z^3}{1 + Z^2 + Z^3} \right). \quad (14)$$

We have computed the bit error rate (BER) and the frame error rate (FER) using the Monte Carlo method as a function of  $E_b/N_0$ . The results over AWGN channels are plotted in Fig. 12: (i) the proposal; (ii) the conventional-A ( $\pi_1$  is used and

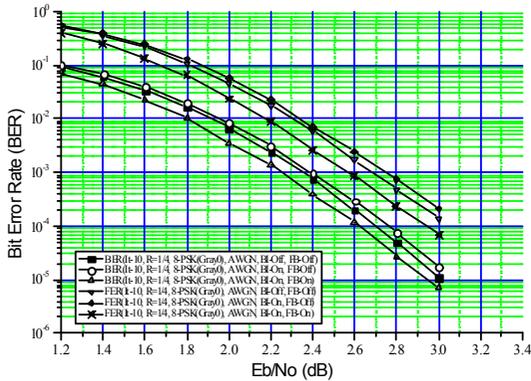


Fig. 10. BER and FER Comparison for SCCC-TCM under AWGN Channel. Data Block Size: 318, Overall Code Rate: 1/4

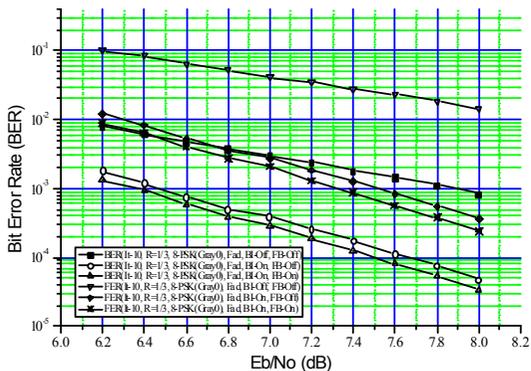


Fig. 11. BER and FER Comparison for SCCC-TCM under Rayleigh Channel. Data Block Size: 318, Overall Code Rate: 1/3

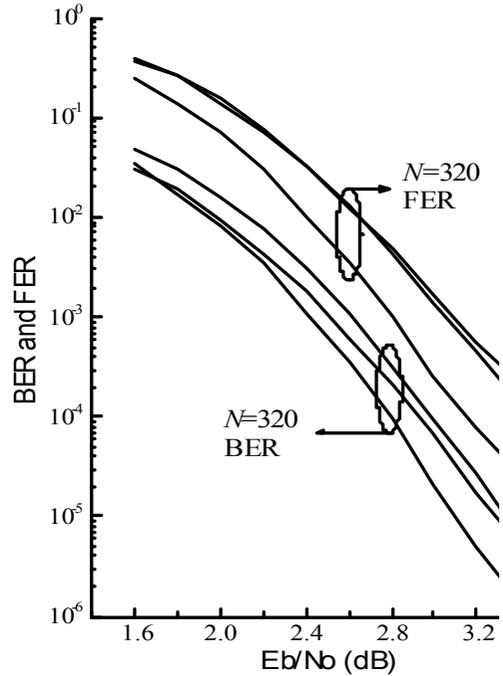


Fig. 12. Performance of PC-TCM (Gaussian channel, rate 1/3 PCCC, 8-PSK)

B-Module is not used); (iii) the conventional-B (Both  $\pi_1$  and B-Module are not used). For (i)-(iii), ten decoding iterations are performed.

As shown in Fig. 12, the proposed schemes achieve better performance. On AWGN channels, the proposed algorithm outperforms the conventional-A by 0.2-0.4 dB and the conventional-B by 0.1-0.2 dB at the BER of  $10^{-5}$  and at the FER of  $10^{-4}$ .

#### IV. Conclusion

In this paper, we considered the enhanced iterative decoding schemes for parallel and serially concatenated trellis coded modulation. Through the simulation results over AWGN channels and Rayleigh fading channels, it has been found that the proposed schemes have outperformed the conventional Turbo decoding schemes. The high performance of the proposed schemes is essentially due to the fact that the extrinsic information not

used in previous schemes is additionally utilized by feedback.

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