Web Tension Control Using Output Feedback

Seungrohk Oh*★

Abstract

We consider a web transport system. The objective of this paper is to design the output feedback controller such that the controller can track a desired tension and processing speed on web transport system. We propose the new design method using observer and feedback linearization technique. The proposed method use a nonlinear feedback to transform to linear system and high gain observer to estimate the state value. We show that the proposed controller can achieve the control object using only output. We show a performance of controller via the simulation.

Key words: web transport system, output feedback, observer, nonlinear feedback, tension control

I. Introduction

The web tension and processing control is important to control the quality of product. The work [1,2,3,4] is to design a controller to control the tension and processing speed. The gain scheduling method[1,2] was used for the control of tension and process speed. Gain scheduling method has a critical disadvantage such that system could ba an unstable when an operating point is changed. The work[1] fixed the processing speed with some operating condition to handle a nonlinearity in the web process dynamics. After fixing the processing speed, the system was changed to a linear system. The linear controller was designed to control the tension and process speed of a web transport system. This approach could cause a problem when the operation condition is changed. To avoid this problem, the work[5] used the nonlinear feedback to transform to a linear system. It is shown that the web transport system can be transformed to a linear system by cancelling the nonlinearity using feedback after the change of coordinate. After transformation to a linear system using the change of coordinate, we design a controller to achieve the control objectives. The method[5] show that the state feedback controller can be used on the every operation condition, since we do not fix the system parameters to remove the nonlinearity existed in the system dynamics. We expand our previous work[5] that system have a modeling uncertainty and the use of observer to estimate a state variables. We use a high gain observer[6] which is a robust observer in the presence of modeling uncertainty and nonlinearity in the system. We use the sliding mode control method to achieve the our control object. We show that sliding mode control with high gain observer can control the tension and processing speed using only output. We also demonstrate the performance of our method via a simulation.

* Dept. of Electronics Engineering, Dankook University

★ Corresponding author

II. Modeling of web transport system and linearization

2.1 System modeling

The web transport system to be considered in this paper is shown in Fig 1. Motors are used to drive the unwind roller and rewind roller. Idle rollers guide the web to load cell. The load cell is used to measure the tension of web. The tension and velocity control can be done by controlling the velocity of the unwind roller and rewind roller. The tension of the web can be controlled by torque control of the unwind roller′ motor: the unwind roller acts as if a brake against moving web. The speed of web can be controlled by the torque control of the rewind roller′ motor. The dynamic equations for the web transport system in Fig. 1 can be described by the following equation[1,3].

$$\begin{aligned} J_u \frac{dw_u(t)}{dt} &= -B_u w_u + r_u T(t) - \tau_u \\ \frac{dT(t)}{dt} &= -\frac{r_r w_r}{L} T(t) + K[r_r w_r - r_u w_u] \\ J_r \frac{dw_r(t)}{dt} &= -B_r w_r + r_r T(t) + \tau_r \end{aligned} \qquad (1)$$

where $J_u \equiv$ moment of inertia of unwind roll including motor($kg/m/\sec^2$), $J_r \equiv$ moment of inertia of unwind roll including motor ($kg/(m/\sec^2)$), $w_u \equiv$ angular velocity of the unwind roll(rad/sec), $w_r \equiv$ angular velocity of the wind roll(rad/s), $B_u \equiv$ coefficient of viscous friction of unwind roll(kg-m-s/rad), $B_r \equiv$ coefficient of viscous friction of wind roll(kg-m-s/rad), $r_u \equiv$ radius of the unwind roll(m), $r_r \equiv$ radius of the unwind roll(m), $T \equiv$ web tension(kg), $\tau_u \equiv$ torque generated by unwind motor(kg/m), $\tau_r \equiv$ torque generated by wind motor(kg/m), $L \equiv$ total length of web(m), $K \equiv$ spring constant of web(kg/m). Note that we neglected the Coulomb friction and dynamics of idle roller and load cell in the equation (1).

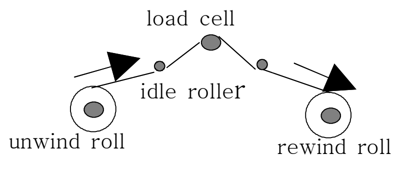

Fig. 1. The web transport system

2.2 Feedback linearization

The main object of this paper is to design the torques generated by unwind roll and rewind roll enabling a desired angular velocity of rewind roll and tension of moving web. The desired processing speed of web can be achieved by controling the angular velocity of the rewind roll. The term w_r of $-r_r w_r/L$ in the equation(1) was fixed at an operating point in the paper[1] because of nonlinearity. After w_r was assumed at the operating condition, the equation (1) is a linear system and one can design the desired controller by using a standard controller design technique for linear system. However the design method used in the paper[1] may cause a problem when operating point is changed. There are some progresses in the nonlinear system theory during the last decade[6,7]. A class of nonlinear system can be transformed to a linear system via a change of coordinate. The class of system is called a feedback linearizable system. To check the feedback linearizability, we rewrite the equation (1) as the following equation.

$$\begin{bmatrix} \dot{w}_u \\ \dot{T} \\ \dot{w}_r \end{bmatrix} = f(w_u, T, w_r) + g \cdot \begin{bmatrix} \tau_u \\ \tau_r \end{bmatrix} \qquad (2)$$

where

$$f(w_u, T, w_r) = \begin{bmatrix} k_1 w_u + k_2 T \\ k_4 w_u + k_5 T w_r + k_6 w_r \\ k_7 T + k_8 w_r \end{bmatrix},$$

$$g = \begin{bmatrix} k_3 & 0 \\ 0 & 0 \\ 0 & k_9 \end{bmatrix}, \qquad k_1 = -\frac{B_u}{J_u},\ k_2 = r_u/J_u,\ k_3 = -1/J_u,$$

$k_4 = -K r_u$, $k_5 = -r_r/L$, $k_6 = K r_r$, $k_7 = -r_r/Jr$, $k_8 = -B_r/J_r$, $k_9 = 1/J_r$. Our interesting outputs are T(t) and w_r. Since $L_{g1} L_f^0 T = 0$, $L_{g1} L_f^1 T = k_3 k_4 \neq 0$,, $L_{g2} L_f^0 T = 0$, $L_{g2} L_f^1 T = (k_5 T + k_6) k_9 \neq 0$, $L_{g1} L_f^0 w_r = 0$, and $L_{g2}^1 L_f^0 w_r = k_9 \neq 0$, where $L_f^1 T$ is the derivative of T along a vector field f, i.e., $L_f^1 T \equiv \begin{bmatrix} \frac{\partial T}{\partial w_u} & \frac{\partial T}{\partial T} & \frac{\partial T}{\partial w_r} \end{bmatrix} \cdot f$, $L_f^0 T \equiv T$, and similarly $L_{gi} L_f^1 T$, $L_{gi} L_f^0 w_r$ are defined where g_i is the i th column vector of the equation (2) has a vector relative degree of [2,1]. Thus implies that the equation (2) is feedback linearizable system. Define $z_1 = T$, $z_2 = k_4 w_u + k_5 T w_r + k_6 w_r$, $z_3 = w_r$. It can be shown that the equation (2) can be rewritten as

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} z_2 \\ b_1(z) + a_{11}(z)\tau_u + a_{12}(z)\tau_r \\ b_2(z) + k_9 \tau_r \end{bmatrix} \qquad (3)$$

where $z = [z_1, z_2, z_3]^T$, $b_2(z) = k_7 z_1 + k_8 z_3$, $a_{11}(z) = k_3 k_4$, $a_{12}(z) = k_5 k_9 z_1 + k_6 k_9$, and

$$b_1(z) = (k_5k_8 - k_1k_5)z_1z_3 + k_5z_2z_3 + k_5k_7z_1^2 + (k_6k_7 + k_2k_4)z_1 + k_1z_2 + (k_6k_8 - k_1k_6)z_3.$$

The equation (3) can be linearize using the feedback. After choosing $\tau_u = \frac{1}{a_{11}}[-b_1(z) - a_{12}(z)\tau_r + v_u]$ and $\tau_r = \frac{1}{k_9}[-b_2(z) + v_r]$ where v_u and v_r are defined in next section.

Ⅲ. Observer and controller design

3.1 High gain observer

In order to reduce the state measurements, the state estimation is needed using a observer. However a standard observer can not use in the nonlinear system. High gain observer can be used to estimate the states for nonlinear systems in the presence of modeling uncertainty[6]. High gain observer can reject the effect of the modeling uncertainty during the state estimation. To construct the high gain observer, let us define the desired web tension be Tr, the desired angular velocity of rewind roll be w_{rr}, and the desired angular velocity of unwind roll be w_{ur}. Note that we assume that Tr, w_{rr}, and w_{ur} are constant. Let tracking error be $x_1 = T - T_r$, $x_2 = k_4(w_u - w_{ur}) + k_5(T - T_r)(w_r - w_{rr}) + k_6(w_r - w_{rr})$, $x_3 = w_r - w_{rr}$. The equation (3) can be rewritten as

$$\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{array} = \begin{array}{c} x_2 \\ b_1(x) + \Delta b_1(x) + a_{11}(x)\tau_u + a_{12}(x)\tau_r \\ b_2(x) + \Delta b_2(x) + k_9\tau_r \end{array}$$

(4)

where $x = [x_1, x_2, x_3]^T$, $b_2(x) = k_7x_1 + k_8x_3$, $a_{11}(z) = k_3k_4$, $a_{12}(x) = k_5k_9x_1 + k_6k_9$, and $b_1(x) = (k_5k_8 - k_1k_5)x_1x_3 + k_5x_2x_3 + k_5k_7x_1^2 + (k_6k_7 + k_2k_4)x_1 + k_1x_2 + (k_6k_8 - k_1k_6)x_3$

Note that we assume there is a modeling uncertainty and denote as $\Delta b_1(x)$ and $\Delta b_2(x)$. Construct the observer as follows:

$$\begin{array}{c} \dot{\hat{x}} \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \end{array} = \begin{array}{c} \hat{x}_2 + (l_1/\epsilon)(x_1 - \hat{x}_1) \\ b_1(\hat{x}) + a_{11}(\hat{x})\tau_u + a_{12}(\hat{x})\tau_r + (l_2/\epsilon^2)(x_1 - \hat{x}_1) \\ b_2(\hat{x}) + k_9\tau_r + (l_3/\epsilon)(x_1 - \hat{x}_1) \end{array}$$

(5)

where ϵ is small enough number and a design parameter and we will specify later on, and l_1 and l_2 are chosen such that the roots of $s^2 + l_1s + l_2 = 0$ has a negative real part and l_3 is some positive number. Define the state estimation error $e = x - \hat{x}$ where $\hat{x} = [\hat{x}_1, \hat{x}_2, \hat{x}_3]^T$. Using the equation (4) and (5), it can be shown that

$$\dot{e}_1 = e_2 - (l_1/\epsilon)e_1$$
$$\dot{e}_2 = b_1(x) - b_1(\hat{x}) + \Delta b_1(x) + [a_{11}(x) - a_{11}(\hat{x})]\tau_u + [a_{12}(x) - a_{12}(\hat{x})]\tau_r - (l_1/\epsilon^2)e_1$$
$$\dot{e}_3 = b_2(x) - b_2(\hat{x}) + \Delta b_2(x) - (l_3/\epsilon)e_3$$

(6)

where $e_i = x_i - \hat{x}_i$, $i = 1,2,3$.

It can be shown that $(1/\epsilon)e_1$, e_2, and $(1/\epsilon)e_3$ becomes oder of ϵ after short period of time with a globally bounded control input[6]. Thus means that specially scaled observer given in the equation (5) can achieve the estimation error e(t) to be oder of ϵ after the short period of time.

3.2 Controller design

Our object is to design a globally bounded control input such that $(x, \hat{x}) \rightarrow 0$ as $t \rightarrow \infty$. To do this end, we chose the sliding surface $s_1 = m_1\hat{x}_1 + \hat{x}_2$, $s_2 = \hat{x}_3$ where m_1 is some positive number. We need to deign τ_u and τ_r such that sliding mode condition, $s_1\dot{s}_1 \leqq 0$ and $s_2\dot{s}_2 \leqq 0$, is satisfied provided estimation error is oder of ϵ. Consider the control input

$$\tau_r = -\frac{1}{k_9}[b_2(\hat{x}) + (\rho_2(\hat{x}) + a2)sgn(s_2)]$$

$$\tau_u = -\frac{1}{a_{11}}[m_1\hat{x}_2 + b_1(\hat{x}) + a_{12}\tau_r + (\rho_1(\hat{x}) + a1)sgn(s_1)] \quad (7)$$

where $\rho_1(x) \geqq \Delta b_1(x)$, $\rho_2(x) \geqq \Delta b_2(x)$, a1 and a2 are some positive constants. Using the equation (5) and control input (7), it can be shown that

$$\begin{aligned} s_1\dot{s}_1 &= s_1[(l_1/\epsilon)e_1 + (l_2/\epsilon^2)e_1 - (\rho_1(\hat{x}) + a1)sgn(s_1)] \\ &= -(\rho_1(\hat{x}) + a1)|s_1| + [(l_1/\epsilon)e_1 + (l_2/\epsilon^2)e_1]s_1 \end{aligned}$$

To show $s_1\dot{s}_1 \leqq 0$, we need to find estimate $(l_2/\epsilon^2)e_1$, while $(l_1/\epsilon)e_1$ is oder of ϵ. To do this end, define $h1 = (1/\epsilon)e_1 + (\epsilon/l_2)\Delta b_1(\hat{x})$. $(l_2/\epsilon^2)e_1 = (h1/\epsilon)l_2 - \Delta b_1(\hat{x})$. It can be shown that $(h1/\epsilon)l_2$ is oder of ϵ as the time progresses. Therefore

$$s_1\dot{s}_1 \leqq -c1|s_1| \quad (8)$$

where c1 is some positive number such that c1<a1. Similarly, we can show that

$$s_2\dot{s}_2 \leqq -c2|s_2| \quad (9)$$

where c2 is some positive number such that c2<a2. The equations (8) and (9) imply that There exists a finite time that trajectory reach the sliding surface and remain thereafter. To show $(x, \hat{x}) \to 0$ as $t \to \infty$, we consider the Lyapunov function $V(\hat{x}_1, \bar{e}) = -(1/2m_1)\hat{x}_1^2 + \bar{e}^T P_1 \bar{e}$ where a positive definite matrix P_1 satisfies $P_1A + A^TP_1 = -I$, $A = \begin{bmatrix} -l_1 & 0 & 0 \\ -l_2 & 0 & 0 \\ 0 & 0 & -l_3 \end{bmatrix}$ and $\bar{e} = [e_1/\epsilon \ e_2 \ e_3]^T$. Note that since A is Hurwitz matrix, there exist a positive definite matrix. The derivative of V along the trajectory satisfies following inequality.

$$\begin{aligned} \dot{V} &\leqq -\hat{x}_1^2 - (1/\epsilon)\|\bar{e}\|^2 + a3|\hat{x}_1|\,\|\bar{e}\| + a4\|\bar{e}\|^2 \\ &\leqq -\begin{bmatrix}|\hat{x}_1| & \bar{e}\end{bmatrix}\begin{bmatrix} 1 & -a3/2 \\ -a3/2 & 1/\epsilon - a4 \end{bmatrix}\begin{bmatrix}|\hat{x}_1| \\ \bar{e}\end{bmatrix} \end{aligned}$$

where a3 and a4 are some positive constant and independent of ϵ. For sufficiently small ϵ, $\begin{bmatrix} 1 & -a3/2 \\ -a3/2 & 1/\epsilon - a4 \end{bmatrix}$ is a positive definite matrix. Therefore $(\hat{x}_1, \bar{e}) \to 0$ as $t \to \infty$. Thus implies that $(x, \hat{x}) \to 0$ as $t \to \infty$. Therefore the control input (7) can achieve the our objects.

Ⅳ. Simulation Results

We consider the experimental system given in [1] as an example. The experimental system has the following data:
$J_u = J_r = 1.95 \times 10^{-5} kg-m-s^2$, L=0.3m, K=200 kg/m,n $B_u = B_r = 2.533 \times 10^{-5}$kg-m, $r_u = 0.04$ m, $r_r = 0.015$ m.
We consider that the desired tension is $T_r = 0.5kg$ and the desired angular velocity is $w_{rr} = 87.5$ rad/sec initially and then the desired angular velocity is changed to 175 rad/sec. We assume that modeling uncertainty $\Delta b_1(\cdot) = 0.08b_1(\cdot)$ and $\Delta b_2(\cdot) = 0.08b_2(\cdot)$. We select the sliding surfaces $s_1 = \hat{x}_1 + \hat{x}_2$ and $s_2 = \hat{x}_3$. We also select observer gain $l_1 = 9$, $l_2 = 20$, and $l_3 = 4$ with $\epsilon = 0.01$. One can verify that $s^2 + l_1s + l_2 = 0$ has a negative real part of roots with $l_1 = 9$ and $l_2 = 20$. For the control input, we choose $\rho_1 = 0.1|b_1(\hat{x})|$, $\rho_2 = 0.1|b_2(\hat{x})|$, a1=0.01, and a2=0.01 in the equation (7). We saturate the control input τ_u and $\tau_r$over 0.02 to be a globally bounded control. The time profile of tension of the web is followed to 0.5 kg as in the Fig. 2. The time profile of angular velocity of rewind web is also well followed 87.5 rad/sec which is the desired one initially in Fig. 3. After changing the desired angular velocity to 175 rad/sec, the time profile of angular velocity of rewind web is well followed 175 rad/sec in Fig. 3.

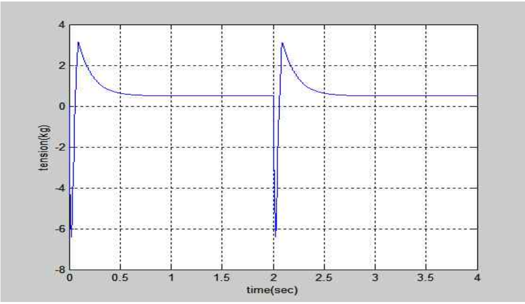

Fig. 2. The plot of tension of web with proposed method

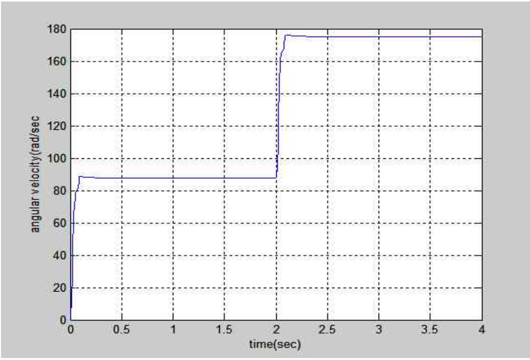

Fig. 3. The plot of angular velocity of rewind roll with proposed method

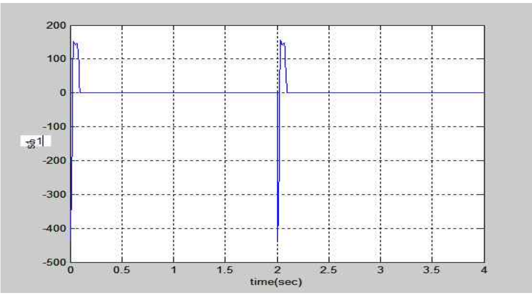

Fig. 4. The value of sliding face s_1

Fig. 4 is the plot of value of sliding surface s_1. One can observe that the sliding mode condition is not satisfied during short period of time. However sliding mode condition is satisfied after short period time, since we design the control input such that sliding mode is satisfied when error between states and estimate of states is oder of ϵ. We simulated the another controller using linearization technique which is used in [1]. After fixing $w_{rr} = 87.5$ rad/sec in equation (1), we consider the same system without modeling uncertainty for the comparison purpose. we design the standard linear state feedback controller such that the closed loop eigen value is equal to [−3, −4, −5]. Fig. 5 is the simulation results. One can observe that the tension is well followed the desired value during the first 5 second. However the tension is not followed the desired value after the first 5 second which is corresponded to the change of the desired angular velocity being 175 rad/sec, since the controller is designed for the system being linearized with $w_r = 87.5$ rad/sec.

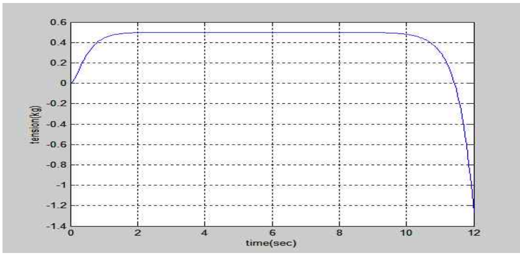

Fig. 5. The plot of tension of web with linearization at $w_r = 87.5$ rad/sec

Ⅴ. Conclusion

We consider a web transport system. We propose the new control design method using high gain observer and feedback linearzation. The proposed controller can work any operating condition, while the previous work[1] restricted to the certain operation condition. We demonstrate that the proposed control design method is working very well under the any operating point, while the previous design method is not working in the different operating point via a simulation. We design the observer based controller under the matching condition being satisfied. We will investigate the case that matching condition is not satisfied.

References

[1] Ku Chin Lin, "Observer-based Tension Feedback Control with Friction and Inertia Compensation," IEEE trans. on control systems technology, 11(1), pp109-118, 2003.

[2] S. H. Song and S.K. Sul, "A New Tension Controller for Continuous Strip Processing Line," IEEE Ind. Application conf., vol3, 1998, pp.2225-2230.

[3] T. Sakamoto and Y. Fujino, "Modeling and Analysis of a Web Tension Control System," in Proc. IEEE Int. Symp., vol. 1, 1997, pp.358-362.

[4] G. E. Youngand K.N. Reid, "Lateral and Logitudinal Dynamic Behavior and Control of Moving Web," J. Dyn. Sys. Meas. Cont., vol. 115, pp. 309-317,1993,.

[5] S. Oh, The design of Web tension control using nonlinear feedback, Journal of KIEE, 55(1), 164-166, 2006

[6] H. K. Khalil. *Nonlinear Systems, 3rd edition.* Prentice Hall, New Jersey, 2002.

[7] A. Isidori, *Nonlinear Control Systems, 3rd ed.,* New York, Springer-Verlag, 1995.

BIOGRAPHY

Seungrohk Oh (Member)

1980 : BS degree in Electrical Engineering, Hanyang University.
1990 : MS degree in Electrical Engineering, Polytechnic University(New York).
1994 : PhD degree in Electrical Engineering, Michigan State University.
1982-1996: Korea Electric Power Cooperation
1996- : Professor, Dankook. Univ.